Invited Paper

Towards improving shell and beam finite elements

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ABSTRACT

We introduce the MITC3+ shell element (Lee et. al, 2014) and the continuum mechanics based beam elements (Yoon and Lee, 2014) recently developed. The elements can be employed for linear and nonlinear analyses of general shell and beam structures with improved performance. The 3-node MITC3+ shell element passes all the basic numerical tests and shows excellent convergence behaviors even in distorted meshes. Indeed, its performance is as good as that of the 4-node MITC4 shell element, which has been widely used in commercial FE software. The continuum mechanics based beam elements can model complicated 3D beam geometries and material compositions by employing cross-sectional discretization, and provides an ability to accurately predict fully coupled 3D behaviors of stretching, bending, shearing, twisting and warping. Its superb modeling capabilities can encompass complicated mechanical characteristics of bio- and nano-structures. In this presentation, we briefly introduce their formulations and demonstrate their performance through numerical examples.

1. INTRODUCTION

Finite element method has been widely used for analysis of shell and beam structures. For a long time, significant efforts have been made to develop more effective and robust structural finite elements.

In this presentation, we share the results of our previous studies on the development of shell and beam finite elements by introducing the MITC3+ shell element (Lee et. al, 2014, 2015, Jeon et. al, 2014, Jeon et. al, 2015) and the continuum mechanics based

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beam elements (Lee and McClure, 2006, 2007, Lee and Noh, 2010, Yoon et. al, 2012, Yoon and Lee, 2014) recently developed. The 3-node MITC3+ shell element passes patch tests, zero energy mode tests, isotropic tests, and shows excellent convergence behavior in linear and nonlinear analyses. Its performance is as good as that of the 4-node MITC4 shell element, which has been widely adopted in most commercial FE software. The continuum mechanics based beam element allows cross-sectional discretization, leading to superb modeling capabilities considering fully coupled 3D behaviors of stretching, bending, shearing, twisting and warping.

In the following sections, the basic formulations of the MITC3 shell element and the continuum mechanics based beam are reviewed and their performances are briefly presented.

2. MITC3+ SHELL ELEMENT

The geometry interpolation of the MITC3+ shell element, shown in Fig. 1, is given by

$$\vec{x}(r,s,t) = \sum_{i=1}^{3} h_i(r,s)\vec{x}_i + \frac{t}{2}\sum_{i=1}^{4} a_i f_i(r,s)\vec{V}_n^i \quad \text{with} \quad a_4\vec{V}_n^4 = \frac{1}{3}(a_1\vec{V}_n^1 + a_2\vec{V}_n^2 + a_3\vec{V}_n^3), \tag{1}$$

in which $f_i(r,s)$ are two-dimensional interpolation functions that include the cubic bubble function f_4 corresponding to the internal node 4

$$f_1 = h_1 - \frac{1}{3}f_4, \quad f_2 = h_2 - \frac{1}{3}f_4, \quad f_3 = h_3 - \frac{1}{3}f_4, \quad f_4 = 27rs(1-r-s). \tag{2}$$

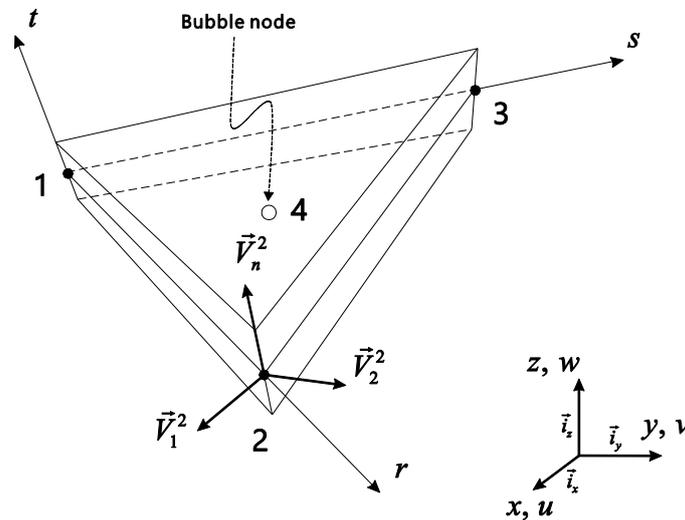
Figure 1. Geometry interpolations of the MITC3+ shell element.

From Eq. (1), we obtain the displacement interpolation

$$\vec{u}(r,s,t) = \sum_{i=1}^{3} h_i(r,s)\vec{u}_i + \frac{t}{2}\sum_{i=1}^{4} a_i f_i(r,s)(-\vec{V}_2^i\alpha_i + \vec{V}_1^i\beta_i), \tag{3}$$

in which α_4 and β_4 are the rotation degrees of freedom at the bubble node.

The assumed transverse shear strain field is used for the MITC3+ shell element, see Fig. 2 for the tying positions used.

$$\hat{e}_{rt} = \frac{2}{3}(e_{rt}^{(B)} - \frac{1}{2}e_{st}^{(B)}) + \frac{1}{3}(e_{rt}^{(C)} + e_{st}^{(C)}) + \frac{1}{3}\hat{c}(3s-1),$$

$$\hat{e}_{st} = \frac{2}{3}(e_{st}^{(A)} - \frac{1}{2}e_{rt}^{(A)}) + \frac{1}{3}(e_{rt}^{(C)} + e_{st}^{(C)}) + \frac{1}{3}\hat{c}(1-3r). \tag{4}$$
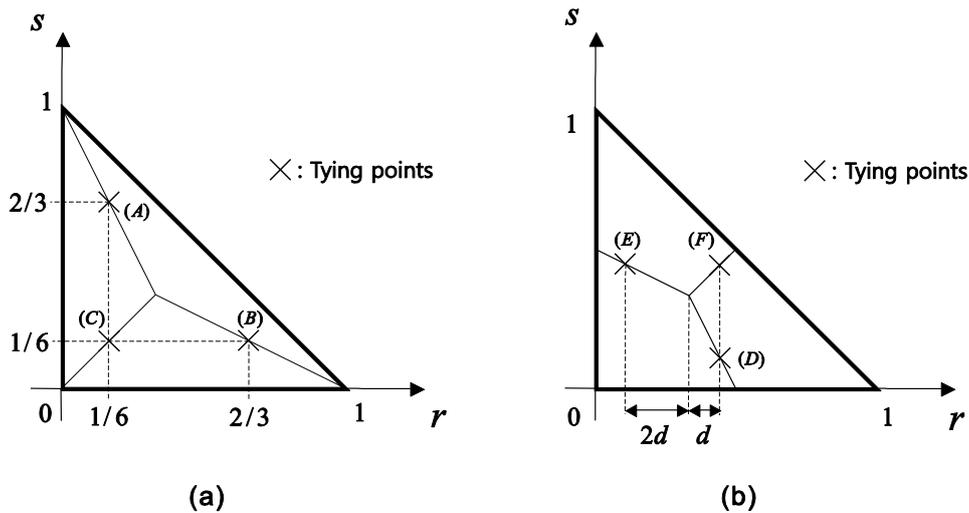

(a)

(b)

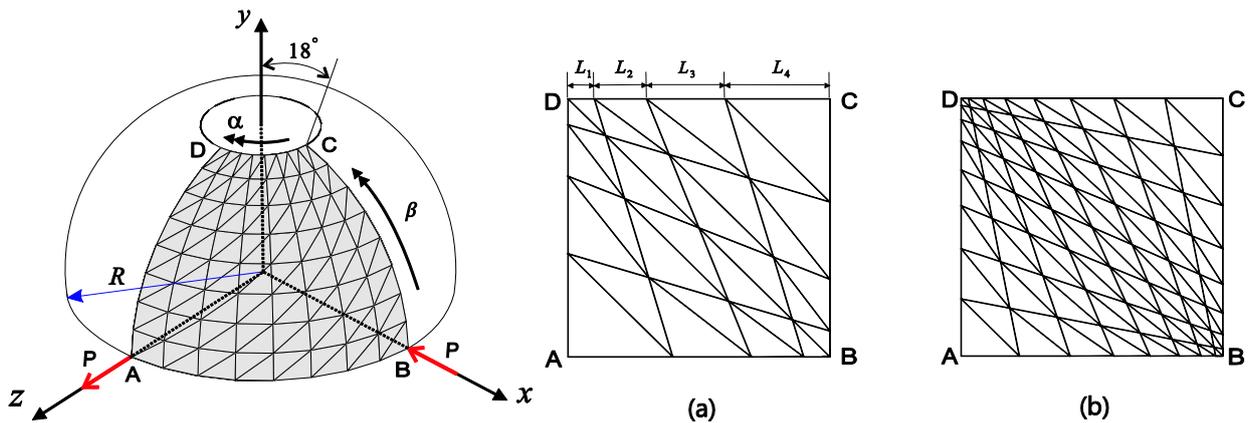Figure 2. Tying positions (A)-(F) for the assumed transverse shear strain field of the shell element.

Figure 3. Hemispherical shell problem ($R=10$, $t=0.04$, $p=1$, $E=6.825\times10^7$ and $v=0.3$) and distorted mesh patterns (a) for $N=4$ and, (b) for $N=8$.

We consider the hemispherical shell problem shown in Figure 3 (Macneal and Harder, 1985). Due to symmetry, only a one-quarter model is considered. The symmetry

condition is imposed: $u_z = \beta = 0$ along BC, $u_x = \beta = 0$ along AD. Both uniform and distorted meshes are considered. Figure 4 shows the convergence curves of the displacement u_z at Point *A* for the MITC3 and MITC3+ shell elements. Unlike the MITC4 shell element, the MITC3+ shell element presents good results even in the distorted mesh cases.
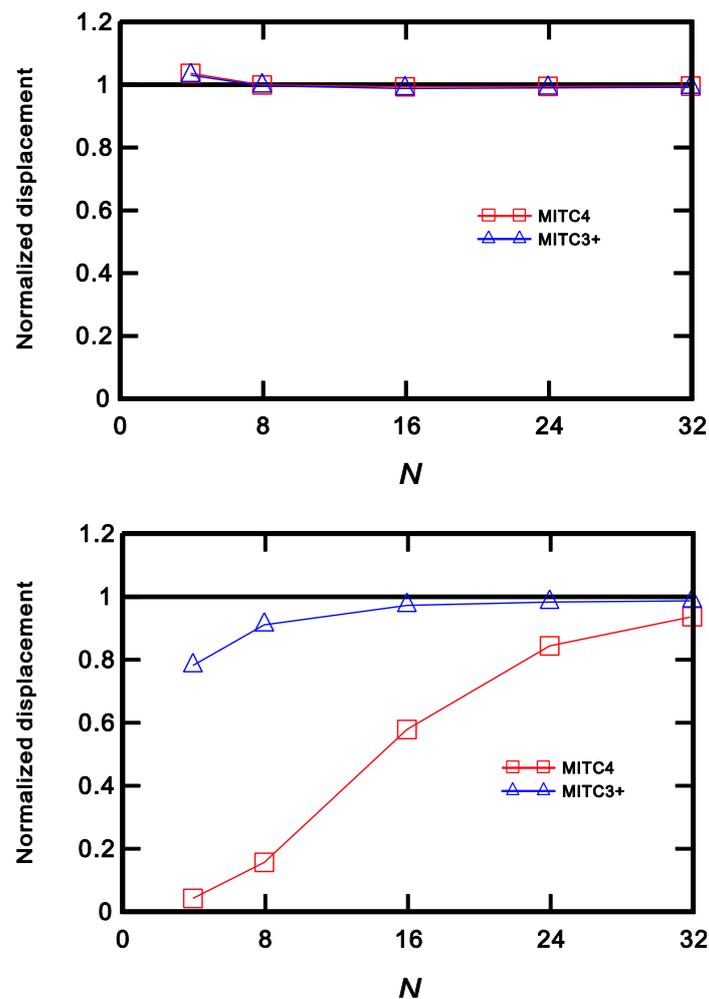
Figure 4. Convergence curves for the hemispherical shell problem.

3. CONTINUUM MECHANICS BASED BEAM

The geometry interpolation of the q-node continuum mechanics based beam finite element for sub-beam m is given by

$$\vec{x}^{(m)} = \sum_{k=1}^{q} h_k(r)\vec{x}_k + \sum_{k=1}^{q} h_k(r)\bar{y}_k^{(m)} \vec{V}_{\bar{y}}^{k} + \sum_{k=1}^{q} h_k(r)\bar{z}_k^{(m)} \vec{V}_{\bar{z}}^{k}$$

$$(5)$$

with $\quad \bar{y}_k^{(m)} = \sum_{j=1}^{p} h_j(s,t)\bar{y}_k^{j(m)} \quad, \quad \bar{z}_k^{(m)} = \sum_{j=1}^{p} h_j(s,t)\bar{z}_k^{j(m)} \quad,$ $\qquad (6)$

where $\bar{y}_k^{(m)}$ and $\bar{z}_k^{(m)}$ denote the material position of the sub-beam element m in the cross-sectional Cartesian coordinate system on cross-sectional plane k. Eq. (5) indicates that the material position on the cross-sectional plane is interpolated by cross-sectional nodes, see Figure 5.

From the interpolation of geometry in Eq. (5), the interpolation of displacements corresponding to the sub-beam m is derived

$$\vec{u}^{(m)} = \sum_{k=1}^{q} h_k(r)\vec{u}_k + \sum_{k=1}^{q} h_k(r)\bar{y}_k^{(m)}\left\{\vec{\theta}_k \times \vec{V}_{\bar{y}}^{k}\right\} + \sum_{k=1}^{q} h_k(r)\bar{z}_k^{(m)}\left\{\vec{\theta}_k \times \vec{V}_{\bar{z}}^{k}\right\} + \sum_{k=1}^{q} h_k(r)f_k^{(m)}(s,t)\vec{V}_r^{k} \quad, \qquad (7)$$

in which $\vec{u}_k$ and $\vec{\theta}_k$ are the displacement and rotation vectors, respectively, in the global Cartesian coordinate system at beam node k, $\vec{u}_k = \left\{u^k \quad v^k \quad w^k\right\}^T$ and $\vec{\theta}_k = \left\{\theta_x^k \quad \theta_y^k \quad \theta_z^k\right\}^T$, and $f_k^{(m)}$ and ${}^t\alpha_k$ are the warping function and the corresponding warping degree of freedom at beam node k.
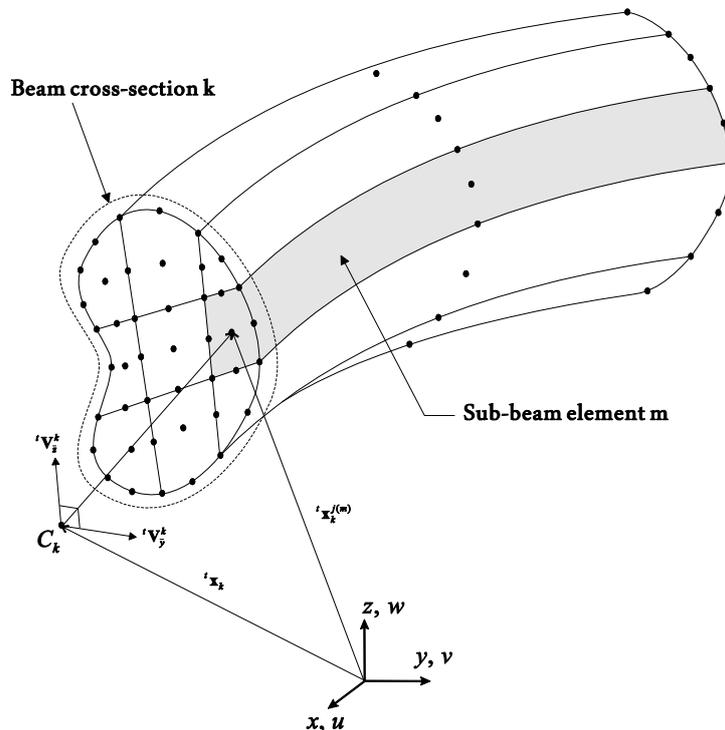

Figure 5. The concept of the continuum mechanics based beam finite element with sectional discretization

The example in Figure 6 illustrates single turn of the double helix DNA geometries and

its continuum mechanics based beam model. Four turns model of length $28.56\,nm$ is discretized by 32 beam elements. The linear elastic material with Young's modulus $E = 50000\,pN/nm^2$ and zero Poisson's ratio is used. Table 1 displays the mechanical properties of proposed DNA continuum model compared with experimental values. Excellent modeling capability and potential of an efficient analysis tool for DNA nanostructures are well illustrated.

(a)   (b)

Figure 6. Double helical DNA problem: (a) DNA structure, (b) beam model. (unit: nm)

Table 1. Comparison of mechanical properties.

	Experimental value (Castro et. al 2011)	Present beam model
Stretching modulus	$1100\,pN$	$1050\,pN$
Bending modulus	$230\,pNnm^2$	$211.4\,pNnm^2$
Twisting modulus	$460\,pNnm^2$	$488.6\,pNnm^2$

4. CONCLUSIONS

In this presentation, we introduced the MITC3+ shell element and the continuum mechanics based beam elements recently developed. Their formulations were briefly reviewed and their performances were presented through representative numerical examples. Both structural finite elements have promising accuracy and modeling capabilities. We believe that both elements will be widely utilized to enhance the predictive ability of finite element models in various engineering practices including design of classical structures and applications of nano- and bio-structures.

ACKNOWLEDGEMENTS

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